A note on public debt sustainability under secular stagnation

1. **Standard sustainability ($i_t > g_t$)**

Standard debt sustainability abstracts from maturity structure by relating the public debt stock at time *t* (b_t) to the previous period stock (b_{t-1}) adjusted for one-period nominal interest (i_t) and growth (g_t) less the primary surplus:

$$b_t = -s_t^P + \left(\frac{1+i_t}{1+g_t}\right) b_{t-1} \qquad (*)$$

To solve for "sustainability" we rearrange in terms of b_{t-1} as:

$$b_{t-1} = \left(\frac{1+g_t}{1+i_t}\right) s_t^P + \left(\frac{1+g_t}{1+i_t}\right) b_t$$

when $i_t > g_t$ the next period is discounted from the perspective of *t-1* which reflects the fact that investors have given up purchasing power today, requiring a positive growth-adjusted interest rate. This therefore discounts the value of the next period primary surplus.

While messy, we can iterate forward *N* periods as:

$$b_{t-1} = \left(\frac{1+g_t}{1+i_t}\right) s_t^P + \left(\frac{1+g_t}{1+i_t}\right)\left(\frac{1+g_{t+1}}{1+i_{t+1}}\right) s_{t+1}^P + \left(\frac{1+g_t}{1+i_t}\right)\left(\frac{1+g_{t+1}}{1+i_{t+1}}\right)\left(\frac{1+g_{t+2}}{1+i_{t+2}}\right) s_{t+2}^P + \cdots +$$
$$\left(\frac{1+g_t}{1+i_t}\right)\left(\frac{1+g_{t+1}}{1+i_{t+1}}\right)\left(\frac{1+g_{t+2}}{1+i_{t+2}}\right) \dots \left(\frac{1+g_{t+N}}{1+i_{t+N}}\right) s_{t+N}^P + \left(\frac{1+g_t}{1+i_t}\right)\left(\frac{1+g_{t+1}}{1+i_{t+1}}\right)\left(\frac{1+g_{t+2}}{1+i_{t+2}}\right) \dots \left(\frac{1+g_{t+N}}{1+i_{t+N}}\right) b_{t+N}$$

which can be simplified as:

$$b_{t-1} = NPV_{t-1}(s_t^P, s_{t+1}^P, s_{t+2}^P, \dots, s_{t+N}^P) + NPV_{t-1}(b_{t+N})$$

Or, the debt stock at time *t-1* is rendered sustainable in a forward looking way, by being supported by some present value of future primary surpluses *plus* the present value of the debt stock rolled into period *t+N*. Since $i_t > g_t$ for $\forall t$ a No Ponzi Condition is necessary, requiring the stock of debt cannot grow faster than the growth adjusted interest rate. If not, the interest could be capitalized continuously and the debt never meaningfully "serviced." Otherwise investors should not hold the debt. The NPC requires that the terminal value of government debt should be non-positive $\lim_{N\to\infty} NPV_{t-1}(b_{t+N}) < 0$, which we set to zero.

Notice how, unlike a household or most corporates, the debt is not repaid but rolled over continuously—an advantage of being a sovereign state. This means for $i_t > g_t$ the stock of debt today is sustainable if it is equal to the present value of future primary surpluses over an infinite horizon:

$$b_{t-1} = \lim_{N\to\infty} NPV_{t-1}(s_t^P, s_{t+1}^P, s_{t+2}^P, \dots, s_{t+N}^P)$$

Government debt can be thought of analytically as a perpetual bond, therefore. Primary surpluses are like "coupons," they are the cash flow to investors that makes the bond worth holding. If these coupons are too small, the face value of debt is too high relative to the present value of cash flows. Either there needs to be fiscal adjustment, sometimes forced by the threat of redemptions by investors (because the debt is not really perpetual) or some restructuring is needed.

If we assume a steady state for variables, dropping the *t* subscript and solving, we alternatively get:

$$b_{t-1} = \left(\frac{1+g}{i-g}\right)\left(1 - \left(\frac{1+g}{1+i}\right)^{N+1}\right)s^P + \left(\frac{1+g}{1+i}\right)^{N+1} b_{t+N}$$

Once again, "sustainability" involves the present value of primary surpluses for *N* periods plus the present value of debt at *t+N*, itself supported by as yet contemplated future primary surpluses. Thus, b_{t-1} can be written as an arithmetically weighted average of primary surpluses and future debt:

$$b_{t-1} = (1-\theta(N))\left(\frac{1+g}{i-g}\right)s^P + \theta(N)b_{t+N} \qquad (**)$$

where $\theta(N) = \left(\frac{1+g}{1+i}\right)^{N+1}$ which approaches zero as $N \to \infty$. Taking infinite horizon limits, we can rearrange for s^P as:

$$s^P = \left(\frac{i-g}{1+g}\right)b_{t-1} \qquad (***)$$

Now this happens to be the same expression for the primary balance needed to stabilize the debt stock at a particular historical value; i.e., rearranging (*). Or, maintaining a constant debt-to-GDP ratio is consistent with debt sustainability. But not simply because debt is not increasing, but more fundamentally because, for steady-state values of key macro variables, the present value of future primary surpluses *at the existing policy configuration* equals the debt stock at the beginning of the period.

As an aside, there are occasions when, in the IMF's debt sustainability template, a gently increasing stock of public debt is shown (with $i_t > g_t$), perhaps plateauing at the end of the horizon. It is still argued in such cases that debt is "sustainable." But this relies on barely-legislated primary balance adjustment (see South Africa Request for Purchase Under Rapid Financing Instrument whereby "debt sustainability critically hinges on sustained implementation of policies to address underlying fiscal and structural weaknesses.") While all sustainability relies on some future policy configuration, assuming the policy will adjust to the arc of sustainability is not a baseline—and should not be passed as such and ought not be labelled sustainable in an analytical sense. This was also the case in Greece, of course.

2. Secular stagnation ($i_t < g_t$)

Risking some abuse of language, let's define secular stagnation as synonymous the case where $i_t < g_t$. In this case, $\theta(N) > 1$ and $\lim_{N\to\infty}\theta(N) = \infty$ in (**). The future is not discounted in an exponentially declining way, instead the further ahead is some future primary surplus or debt stock, the higher the weight in the intertemporal budget constraint. In this case, while the maths in (**) continues to apply in finite time, there is no infinite horizon limit.

Where to next?

Well, imagine that the period of secular stagnation lasts for only N periods. Then (**) can still be rearranged for the primary surplus as:

$$s^P = -\frac{1}{\theta(N)-1}\left(\frac{i-g}{1+g}\right)b_{t-1} + \frac{\theta(N)}{\theta(N)-1}\left(\frac{i-g}{1+g}\right)b_{t+N}$$

The "sustainable" primary surplus today is composed of two terms. The first, similar to (***), depends only on the historical debt stock. Note how this still requires a primary surplus since $\theta(N) > 1$ and $i < g$ given the negative pre-multiplier. The second is the debt stock entering the period when secular stagnation ends—i.e., is entirely forward looking to b_{t+N}. This latter term argues for a primary deficit today. The true sustainable primary balance is some trade-off between the two, therefore, depending on the time secular stagnation will last.

This expression can be rearranged as:

$$s^P = \frac{1}{\theta(N)-1}\left(\frac{i-g}{1+g}\right)[b_{t+N} - b_{t-1}] + \left(\frac{i-g}{1+g}\right)b_{t+N} \qquad (\wedge)$$

Once again, the focus in terms of sustainability is on the future sustainable debt stock (b_{t+N}), or the debt which can be rendered sustainable by policies once the period of secular stagnation comes to an end. Since $i < g$ a primary deficit is sustainable both if $b_{t+N} > b_{t-1}$, from the first term on the right, and from the second. The first means the future sustainable debt is greater than that inherited today, meaning fiscal policy can facilitate movement towards this future stock during secular stagnation. Otherwise a surplus may be required.

Of course, as $N \to \infty$ then the first term in (^) approaches zero, in which case we are left with the secular stagnation analogue of the (***) which is that

$$s^P = \left(\frac{i-g}{1+g}\right)b_{t+N} \qquad (\wedge\wedge)$$

Now, the primary balance does not depend on the bequeathed public debt stock at all, only on the future stock. So, there is no need to worry about stabilizing the backward-looking stock. Instead the focus should be the sustainable stock at some point in the unknown future when secular stagnation comes to an end.

One does not need to rely on secular stagnation being likely over an infinite horizon for (^^) to be a good approximation, however. Instead, so long as N is large enough, the first term will be second order providing $b_{t+N} \approx b_{t-1}$, $i \approx g$ in which case focusing on (^^) alone will be reasonable.

Of course, from period N onwards a new fiscal regime will be necessary to assure that public debt is sustainable. Using the traditional argument, with overbars representing variables in the steady state thereafter, we have:

$$b_{t+N} = \left(\frac{1+g}{\bar{\imath}-g}\right) \bar{s}^P$$

where the nominal growth rate is assumed unchanged but where $\bar{\imath} > g$ and the appropriate NPC applied.

Substituting into (^^) we now get the sustainable primary balance in terms of the future post-secular stagnation primary balance:

$$s^P = \left(\frac{i-g}{\bar{\imath}-g}\right) \bar{s}^P$$

Let $\delta = \bar{s}^P - s^P$ represent feasible primary balance adjustment during the transition between secular stagnation and normalization (an adjustment that could last many years). Then:

$$\delta = \bar{s}^P - s^P = \bar{s}^P - \left(\frac{i-g}{\bar{\imath}-g}\right) \bar{s}^P = \left(\frac{\bar{\imath}-i}{\bar{\imath}-g}\right) \bar{s}^P$$

Which we can rearrange and substitute into the expression for s^P to give:

$$s^P = (i-g)\left(\frac{\delta}{\bar{\imath}-i}\right) \qquad \text{(^^^)}$$

And so, the sustainable primary balance today is a function of the ratio of the feasible fiscal adjustment to the change in interest rate during normalization. This is pre-multiplied by the $i - g < 0$. Suppose $i = 0$ and $g = 0.04$ while $\bar{\imath} = 0.05$. And suppose a primary balance adjustment of 5% is considered reasonable ($\delta = 0.05$). Then $s^P = -0.04 = -4\%$. So, a primary deficit of 4% for as long as secular stagnation continues is perfectly sustainable because the primary balance adjustment is equal to the expected change in the interest rate during normalization. Of course, if $\bar{\imath} = 0.06$ then the primary deficit would have to be reduced to 5/6*-0.04=-3.3%.

Alternatively, suppose that the normalized interest rate will be only be marginally higher than the future nominal growth rate, or $\delta = 0.045$ while feasible primary balance adjustment is 7%. Then the primary deficit can be 6.2% so long as secular stagnation continues while future adjustment is feasible to deliver sustainability.

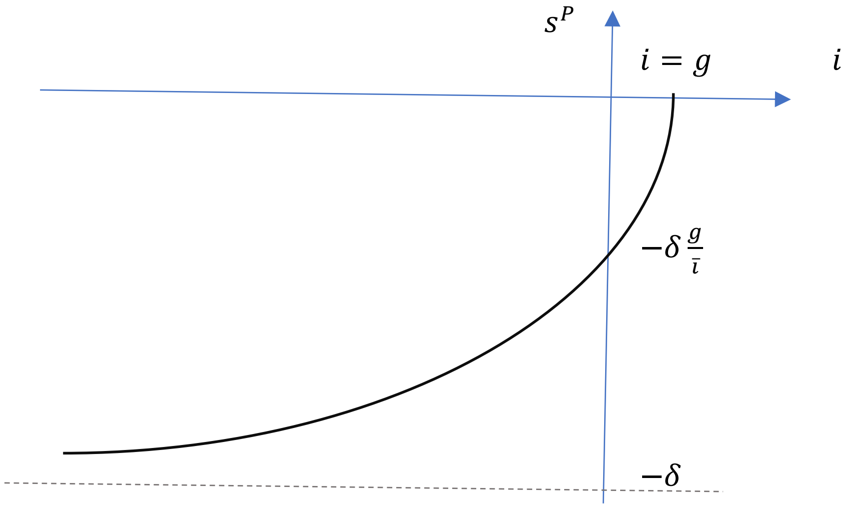
3. What's the relation between s^P and i under secular stagnation?

What else can we say about the primary deficit during the period of secular stagnation? The relationship is clearly non-linear, with i in both the numerator and denominator in (^^^). The slope of the relationship between s^P plotted against i as given by the first two derivatives is:

$$\frac{\partial s^P}{\partial i} = \frac{\delta(\bar{\imath}-g)}{(\bar{\imath}-i)^2} > 0 \qquad \frac{\partial^2 s^P}{\partial i^2} = \frac{2\delta(\bar{\imath}-g)}{(\bar{\imath}-i)^3} > 0$$

meaning there is an upward sloping relation with an increasing gradient. Using L'Hôpital's rule it is already straightforward to show that s^P asymptotes at $-\delta$. We also know that the curve end when $i = g$, beyond which secular stagnation would no longer apply.

Using this, we can sketch s^P as a function of i.



What the sketch above doesn't fully capture is the gradient of the primary balance response to the interest rate being pushed below the nominal growth rate during the early stages of secular stagnation. We know that the primary balance asymptotes to $-\delta$ for $i \to -\infty$. We can ask: what is the value of i for which $s^P = -\delta/2$ is sustainable? The answer is $i = 2g - \bar{\iota}$, or for the parameters used above ($g = 0.04$ while $\bar{\iota} = 0.05$) this means $i = 0.03 = 3\%$. So, the primary balance should move sustainably into deficit of 2.5% for the nominal interest paid falling only 1 percentage point below the nominal growth rate.

More generally, the curve crosses the y-axis at $s^P = -\delta\, g/\bar{\iota}$. That means, when the average nominal interest on government debt reaches $i = 0\%$, which is within touching distance for some euroarea countries, the primary balance deficit should reach 4/5ths the feasible primary balance adjustment for the parameters used here. That means for a 5% feasible primary balance adjustment the primary deficit should run at 4% until secular stagnation comes to an end.

4. Conclusions

Overall, I suspect something as implied by this analysis has been going on in Japan, meaning debt is perfectly sustainable—but traditional sustainability analysis flashes red without any good reason.

This suggests focus in terms of public debt sustainability during secular stagnation should be on the ratio of feasible future primary balance adjustment to the change in average interest rate on debt expected during normalization. I would conjecture that upon careful consideration the former will be larger than the latter, and the potential for large and persistent primary deficits entirely consistent with public debt sustainability.

Chris Marsh, London, November 2020